Solutions to the exercises, specified in the example of the ExSol package

Walter Daems

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Exercise 2-1: Solve the following equation for $x \in C$, with C the set of complex numbers:

$$5x^2 - 3x = 5 \tag{1}$$

Solution: Let's start by rearranging the equation, a bit:

$$\begin{aligned} 5.7x^2 - 3.1x &= 5.3 && (2) \\ 5.7x^2 - 3.1x - 5.3 &= 0 && (3) \end{aligned}$$

The equation is now in the standard form:

$$ax^2 + bx + c = 0 \tag{4}$$

For quadratic equations in the standard form, we know that two solutions exist:

$$x_{1,2} = \frac{-b \pm \sqrt{d}}{2a} \tag{5}$$

with

$$d = b^2 - 4ac \tag{6}$$

If we apply this to our case, we obtain:

$$d = (-3.1)^2 - 4 \cdot 5.7 \cdot (-5.3) = 130.45 \tag{7}$$

and

$$\begin{aligned} x_1 &= \frac{3.1 + \sqrt{130.45}}{11.4} = 1.27 && (8) \\ x_2 &= \frac{3.1 - \sqrt{130.45}}{11.4} = -0.73 && (9) \end{aligned}$$

The proposed values $x = x_1, x_2$ are solutions to the given equation.

Exercise 2-2: Consider a 2-dimensional vector space equipped with a Euclidean distance function. Given a right-angled triangle, with the sides A and B adjacent to the right angle having lengths, 3 and 4, calculate the length of the hypotenuse, labeled C.

Solution: This calls for application of Pythagoras' theorem, which tells us:

$$\|A\|^2 + \|B\|^2 = \|C\|^2 \tag{10}$$

and therefore:

$$\begin{aligned} \|C\| &= \sqrt{\|A\|^2 + \|B\|^2} && (11) \\ &= \sqrt{3^2 + 4^2} && (12) \\ &= \sqrt{25} = 5 && (13) \end{aligned}$$

Therefore, the length of the hypotenuse equals 5.